Localized gap-edge fields of one-dimensional photonic crystals with an ε-negative and a μ-negative defect

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We study the properties of one-dimensional photonic crystals with an ε-negative and a μ-negative defect. With suitable parameters, the pair defect is equivalent to a transparent material with zero effective refractive index. This special pair defect has no influence on the spectral gap formed by the interference of propagating waves in positive-refractive-index materials. However, the field distribution is modified noticeably by the decaying wave in the pair defect. Particularly, the gap-edge field can be a highly localized wave instead of the usual standing wave as the size of the pair defect increases. The localized gap-edge field can reduce the switching thresholds for bistability greatly when Kerr-type nonlinear μ-negative material is involved. A non-ideal model when the ε-negative and μ-negative materials are dispersive and lossy is used to verify the unusual properties.

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I. INTRODUCTION

Recently, photonic crystals containing metamaterials have received special attention for their peculiar properties and potential applications [1–6]. The metamaterials include double-negative materials whose permittivity (ε) and permeability (μ) are simultaneously negative [7,8] and single-negative materials including ε-negative media ($\varepsilon<0$, $\mu>0$) and μ-negative media ($\mu<0$, $\varepsilon>0$) [9–14]. It is known that photonic crystals made of transparent (lossless) positive-refractive-index materials can be opaque to the incident electromagnetic (EM) wave, owing to the appearance of photonic band gaps (PBGs) [15,16]. The PBGs come from the multiple scattering of propagating waves in the periodic structure. On the other hand, the EM waves in single-negative materials are decaying waves since their wave vectors are complex. However, photonic crystals composed of two kinds of opaque single-negative materials can be (perfectly) transparent because of the (resonantly) tunneling of decaying waves [5,6]. Up to now, to the best of our knowledge, there are few works on photonic crystals consisting of transparent positive-refractive-index materials and opaque single-negative materials. It is expected that the transmission properties of this kind of photonic crystal will be special because of the interactions between propagating and decaying waves.

In this paper, we study the properties of one-dimensional (1D) photonic crystals (with positive-refractive-index materials) containing an ε-negative and a μ-negative defect. The pair defect with suitable parameters is equivalent to a transparent material with zero effective refractive index. This special pair defect does not affect the spectral gap induced by the interference of propagating waves in the photonic crystal, as shown in Sec. II. However, the field distribution involving the decaying wave in the pair defect changes noticeably. The field inside the pair defect can be enhanced greatly as the size of the pair defect increases. Accordingly, the gap-edge field becomes a highly localized wave instead of standing wave. In Sec. III, we show that the localized gap-edge field can facilitate the production of bistability when the dielectric constant of the μ-negative material is of Kerr type. In Sec. IV, we confirm that our results still exist when the single-negative materials are dispersive and lossy. Finally, we conclude in Sec. V.

II. LOCALIZED GAP-EDGE FIELDS

We assume that a 1D photonic crystal composed of positive-refractive-index materials is doped by an ε-negative and a μ-negative defect, as schematically shown in Fig. 1. We denote the structure as $(AB)_m CD(AB)_{m-1}A$. A and B indicate positive-refractive-index materials. C and D represent a μ-negative material and an ε-negative material, respectively. m is the period number. The permittivity and permeability of A, B, C, and D layers are supposed to be ε_A, μ_A, ε_B, μ_B, ε_C, μ_C, ε_D, and μ_D, respectively. The thicknesses of A, B, C, and D layers are assumed to be d_A, d_B, d_C, and d_D, respectively.

We suppose a transverse electric wave, e.g., the electric field $\vec{E}$ lying in the y direction as shown in Fig. 1, is normally (along the z direction) incident on the structure. The media in the two sides of the structure are air. The treatment for a transverse magnetic wave is similar. The transmittance of the structure and the field distribution can be obtained by means of the transfer-matrix method [14,17]. In the following calculation, we suppose that $\varepsilon_A=9$, $\varepsilon_B=2$, $\mu_A=\mu_B=1$, $\varepsilon_C=1$, $\mu_C=-1$, $\varepsilon_D=-1$, $\mu_D=1$, $d_A=d_B=18$ mm, and $m=8$. First, we consider the pair defect composed of a μ-negative (C) layer and an ε-negative (D) layer. The pair defect is perfectly transparent in the zero (volume) average permittivity and permeability conditions:

$$\bar{\varepsilon}=\frac{\varepsilon_C d_C+\varepsilon_D d_D}{d_C+d_D}=0,$$

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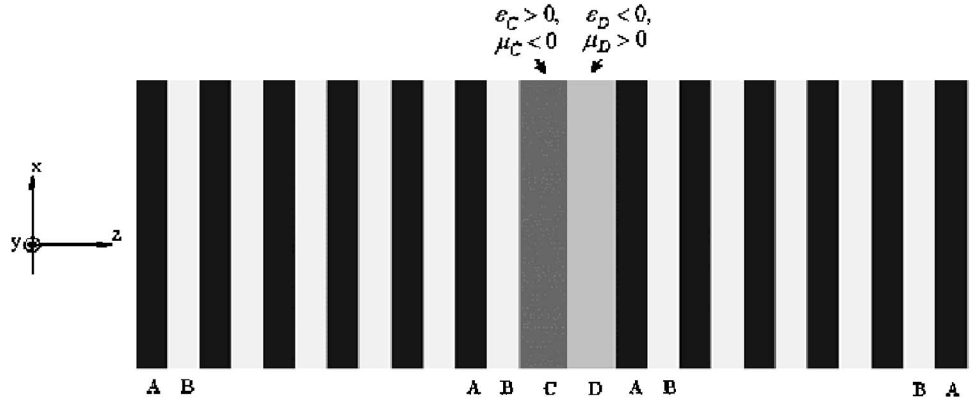

FIG. 1. Schematic of a 1D *AB* photonic crystal with *C* and *D* defects. *A* and *B* denote positive-refractive-index materials. *C* and *D* represent a μ-negative material and an ε-negative material, respectively.

$$\bar{\mu} = \frac{\mu_C d_C + \mu_D d_D}{d_C + d_D} = 0. \tag{1}$$

Equations (1) can be derived from the general result in Ref. [12]. They are equivalent to the impedance matching and effective-phase-shift matching conditions in Ref. [11]. The EM wave can tunnel through the pair defect satisfying Eqs. (1) without any phase delay since the pair defect is reduced to nihility [12]. The tunneling mode is localized at the interface between the two kinds of layers in order to match the boundary condition [6]. Then, we study the properties of a 1D photonic crystal with the embedded pair defect. The variance of the transmittance of the structure with different values of d_C and d_D is shown in Fig. 2. The frequency is the angular frequency in units of gigahertz. Figure 2(a) gives the transmittance of a perfect 1D photonic crystal. A forbidden gap appears because of the interference of propagating waves in *A* and *B* layers. After a pair defect satisfying Eqs. (1) is inserted into the 1D photonic crystal, it is seen that the transmittances of the structures in Figs. 2(b) and 2(c) remain *invariant*, as compared to that in Fig. 2(a). In fact, when Eqs. (1) are met, the pair defect is equivalent to a transparent layer with zero effective refractive index. This special pair defect has no effect on the interference of propagating waves in *A* and *B* layers since the interference comes from the phase difference. However, once deviating from the conditions of Eqs. (1) a lot, the transmittance of the structure will change greatly. The EM wave can hardly transmit through the structure in a wide frequency region, as shown in Fig. 2(d). Actually, $\bar{\varepsilon}=-3/5<0, \bar{\mu}=3/5>0$ when $d_C=12$ mm and $d_D=48$ mm. This means the pair defect in this case is effectively opaque and the EM wave will be blocked.

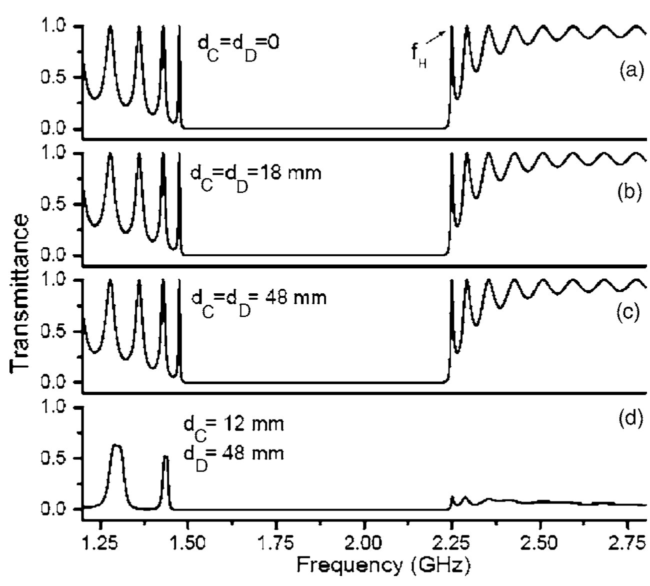

FIG. 2. Transmittances through $(AB)_{15}A$ structure (a) and $(AB)_8CD(AB)_7A$ structures with different values of d_C and d_D (b)–(d). $\varepsilon_A=9$, $\varepsilon_B=2$, $\mu_A=\mu_B=1$, $\varepsilon_C=1$, $\mu_C=-1$, $\varepsilon_D=-1$, $\mu_D=1$, and $d_A=d_B=18$ mm.

Although the transmittance of the structure does not change as long as Eqs. (1) are met, the field configuration indeed changes noticeably because of the decaying wave in the pair defect. We focus on the fields corresponding to the gap-edge frequencies. The high gap-edge frequency in Fig. 2(a) is denoted by f_H. The electric field is denoted by $E(z)$. We suppose the square of the incident electric field is 1 V^2/m^2. In Fig. 3 we calculate the square of the electric fields ($|E(z)|^2$) inside the structures used for Figs. 2(a)–2(c) at frequency f_H. For a perfect 1D photonic crystal, the electric fields corresponding to frequency f_H concentrate their energy in the low-ε regions, as shown in Fig. 3(a). The gap-edge field is a standing wave. In Figs. 3(b) and 3(c) we show the $|E(z)|^2$ inside a 1D photonic crystal with a pair defect for different values of $d_C=d_D$. The gap-edge field is a propagating mode in the photonic crystal while a decaying-wave-based interface mode in the pair defect. It is seen from Figs. 3(a)–3(c) that the squares of the fields in the photonic crystals are the same, regardless of the value of $d_C=d_D$. But the square of the field in the pair defect boosts noticeably when $d_C=d_D$ increases, as shown in Figs. 3(b) and 3(c). The peak value at the center of the pair defect is enhanced more than one order of magnitude when the value of $d_C=d_D$ increases from 18 to 48 mm. In fact, the "enhancement" inside the pair defect with increasing $d_C=d_D$ is the property of the pair defect. In Fig. 4 we calculate $|E(z)|^2$ inside the pair defect surrounded by the air for different values of $d_C=d_D$. It is seen from Fig. 4 that peaking also occurs in the middle of the pair defect as $d_C=d_D$ increases. However, for the same value of $d_C=d_D$, the peak value at the center of the pair defect surrounded by photonic crystals is much higher than that of pair defect surrounded by the air, as seen in Figs. 3(b) and 4(a) [or Figs. 3(c) and 4(b)]. This fact is due to the confinement

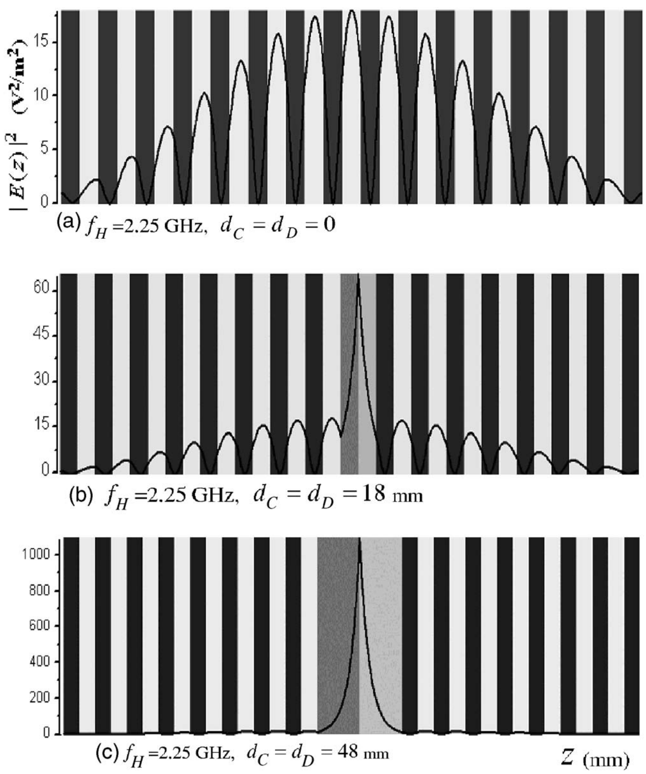

FIG. 3. The square of the electric fields inside the structures used for Figs. 2(a)–2(c) at frequency f_H.

effect of the PBGs of photonic crystals. Therefore, for a 1D photonic crystal with a suitable pair defect, the gap-edge field can evolve into a highly localized wave as the size of the pair defect increases. However, for a 1D conventional photonic crystal, only defect modes in a forbidden gap are localized waves.

III. BISTABLE SWITCHING

It is known that a 1D photonic crystal is a distributed feedback structure if one type of layer has Kerr-type nonlin-

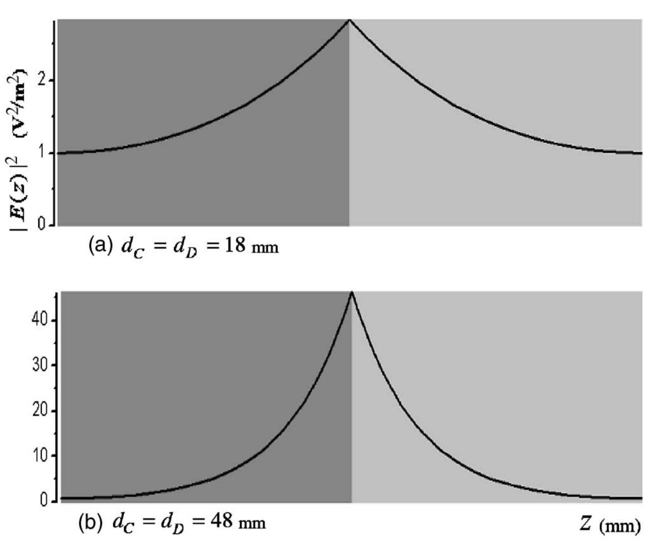

FIG. 4. The square of the electric fields inside the pair defect for frequency at 2.25 GHz.

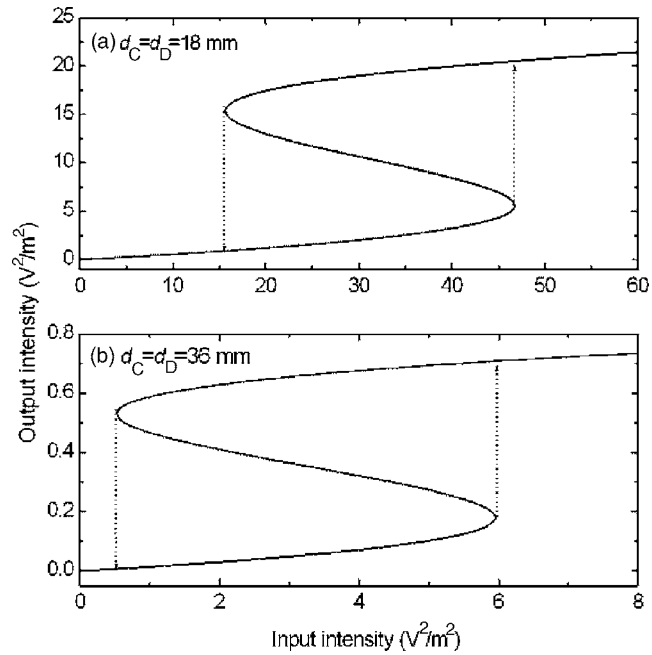

FIG. 5. Output vs input intensity for the structures with different sizes of the pair defect.

earity [18,19]. The gap-edge frequency will shift dynamically since the nonlinear dielectric constant varies with the field, leading to the nonlinear effect such as bistability. For this kind of nonlinear photonic crystal, each one type of layer need has nonlinearity. However, for a 1D photonic crystal with an ε-negative and a μ-negative defect, we only need a nonlinear μ-negative material to alter the gap edge since the corresponding field can be highly localized at the pair defect. We suppose the dielectric constant of the μ-negative material is of Kerr type, that is,

$$\varepsilon_C = \varepsilon_1 + \chi^{(3)}|E(z)|^2. \tag{2}$$

In Eq. (2), ε_1 is a linear dielectric constant, $\chi^{(3)}$ is a nonlinear coefficient. We take $\varepsilon_1 = 1.18$, $\chi^{(3)} = 10^{-4}$ m^2/V^2, $\mu_C = -1.2$, $\varepsilon_D = -1.2$, $\mu_D = 1.2$, and $m = 8$. The parameters of the positive-refractive-index materials are the same as those in Fig. 2. In this section, the input (output) intensity denotes the square of the incident (transmitted) electric field. We treat the nonlinear μ-negative layer by means of a nonlinear transfer-matrix method [20]. Then, we deduce recursively the input intensity from the output intensity. In Fig. 5 we show the output versus input intensity for different values of $d_C = d_D$ when the frequency of the incident wave is 2.24 GHz (around the gap edge). Bistable hysteresis loops caused by the dynamic shifting of the gap edge are both seen in Figs. 5(a) and 5(b). However, the switching thresholds in Fig. 5(b) for $d_C = d_D = 36$ mm are much lower than those in Fig. 5(a) for $d_C = d_D = 18$ mm. The switch-up threshold decreases about one order of magnitude when the size of the pair defect doubles. In Figs. 3(b) and 3(c) we see the gap-edge field is enhanced greatly inside the pair defect as the size of the pair defect increases. Accordingly, this highly localized gap-edge field helps to reduce the switching thresholds for bistability noticeably. Therefore, we can realize very low switching

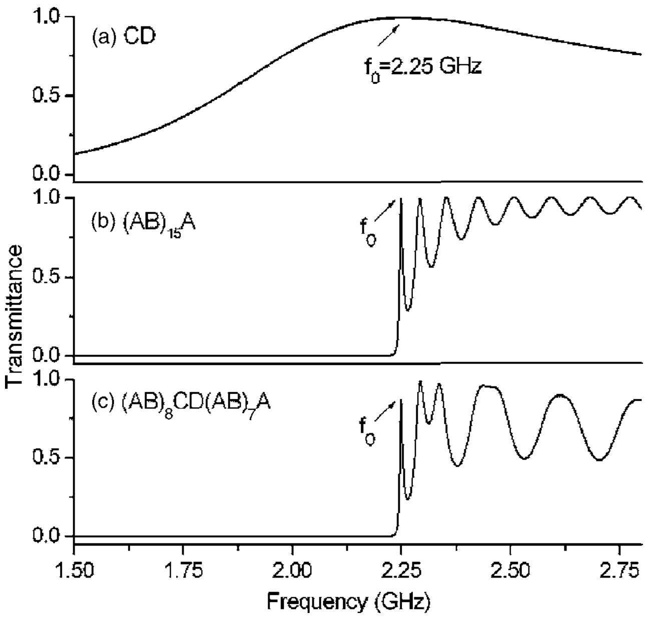

FIG. 6. Transmittances through CD (a), $(AB)_{15}A$ (b), and $(AB)_8CD(AB)_7A$ (c) structures. $\varepsilon_c=\mu_d=1$, $\mu_c=\varepsilon_d=1$, $\alpha=\beta=400$, $\gamma_m=\gamma_e=2\pi\times 3\times 10^6$ s^{-1}, $d_C=d_D=18$ mm. The parameters of A and B are the same as those in Fig. 2.

thresholds just by increasing the size of the pair defect instead of adding the period number as usually done in conventional photonic crystals.

IV. A DISPERSIVE MODEL

In the above calculations, we considered dispersionless single-negative materials. In practice, the negative material parameters should be dispersive. Now, we use the Drude model to describe the single-negative materials, that is,

$$\varepsilon_C=\varepsilon_c, \quad \mu_C=\mu_c-\frac{\alpha}{\omega^2+i\omega\gamma_m}, \tag{3}$$

in a μ-negative material and

$$\varepsilon_D=\varepsilon_d-\frac{\beta}{\omega^2+i\omega\gamma_e}, \quad \mu_D=\mu_d, \tag{4}$$

in an ε-negative material. In Eqs. (3) and (4); $\omega=2\pi f$ is frequency in units of gigahertz and γ_m and γ_e denote the damping. After Eqs. (3) and (4) are substituted into Eqs. (1), the imaginary parts of $\bar{\varepsilon}$ and $\bar{\mu}$ can be neglected if γ_m and γ_e are much smaller than ω. With suitable parameters, we can still find a frequency f_0 satisfying $\bar{\varepsilon}(f_0)=\bar{\mu}(f_0)=0$.

A bilayer (CD) structure made of a μ-negative material and an ε-negative material described by Eqs. (3) and (4) is only transparent in a frequency range. In order to get a wide

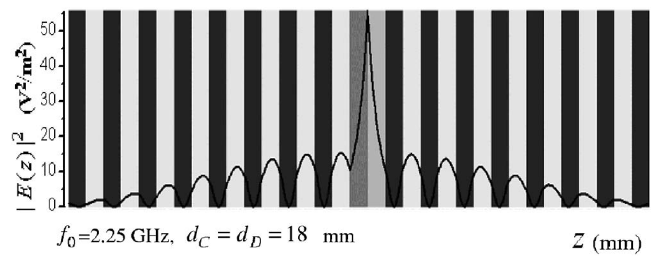

FIG. 7. The square of the electric field inside the structure used for Fig. 6(c) at frequency f_0.

passband, we use $\varepsilon_c=\mu_d=1$, $\mu_c=\varepsilon_d=1$, $\alpha=\beta=400$, $d_C=d_D=18$ mm, and include small losses $\gamma_m=\gamma_e=2\pi\times 3\times 10^6$ s^{-1}. The transmittances through CD, $(AB)_{15}A$, and $(AB)_8CD(AB)_7A$ structures are shown in Figs. 6(a)–6(c). With the chosen parameters, f_0 is 2.25 GHz and just identical to the high gap-edge frequency of the $(AB)_{15}A$ structure. Moreover, the frequency range of the passband in Fig. 6(a) overlaps that of the passband in Fig. 6(b). So the transmittance of the $(AB)_8CD(AB)_7A$ structure is almost the same as that of $(AB)_{15}A$ structure around frequency f_0 except that the transmittance decreases a little, as shown in Fig. 6(c). In Fig. 7 we calculate the square of the electric field inside the structure used for Fig. 6(c) at frequency f_0. The localized gap-edge field is similar to that in Fig. 3(b) though the square of the electric field decreases a little because of the dissipation. Therefore, we verify that the results derived in the dispersionless case still exist when dispersive materials with small losses are used.

V. CONCLUSION

In conclusion, we have studied the transmission properties of 1D photonic crystals with an ε-negative and a μ-negative defect by means of the transfer-matrix method. The transmittance of the structure is independent of the pair defect so long as the zero average permittivity and permeability conditions maintain. With the increase of the size of the pair defect, the gap-edge field evolves into a highly localized wave. Bistability with low switching thresholds is produced by the dynamic shifting of the gap edge when the dielectric constant of the μ-negative material is intensity dependent. The unusual properties are still maintained even when the single-negative materials are dispersive and lossy.

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